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MAGNETIC MOMENT OF THE TWO-PARTICLE BOUND STATE IN QUANTUM ELECTRODYNAMICS[1]

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Abstract

We have formulated the quasipotential method for the calculation of the relativistic and radiative corrections to the magnetic moment of the two-particle bound state in the case of particles with arbitrary spin. It is shown that the g-factors of bound particles contain $O(\alpha^2)$ terms depending on the particle spin. Numerical values for the g-factors of the electron in the hydrogen atom and deuterium are obtained.

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The study of electromagnetic properties of hydrogen-like atoms and ions in quantum electrodynamics (QED) is one of the basic tasks in the theory of two-particle bound states. The experimental verification of the g-factors calculation for the bound particles is carried out during many years [1, 2]. The values of the electron g-factors in hydrogen atom, deuterium, tritium and helium ($^4He^+$), measured in many experiments, are in good agreement with theoretical results. Recently the scope of the experimental investigation of hydrogenic ions was essentially inlarged [3, 4]. These experiments generate a need for the new theoretical study of different contributions to the gyromagnetic factors of the bound particles [5, 6, 7]. At present, the most accurate value for the electron g-factor is obtained in the experiment with the hydrogenic carbon ion $^{12}C^{5+}$ (Z=6) [3, 4]:

$$g_e^{exp}(^{12}C^{5+}) = 2.001\ 041\ 596\ 4(8)(6)(40), \tag{1}$$

where the statistical (8), systematical (6) errors and inaccuracy, connected with the electron mass (40), are shown in brackets. Theoretical investigations of the electromagnetic properties of the hydrogen-like atoms, which were done in [8-14], showed that the gyromagnetic factors of the bound particles can be written in the from:

$$g(H - \text{atom}) = 2 + \Delta g_{rel} + \Delta g_{rad} + \Delta g_{rec} + ... \tag{2}$$

Relativistic corrections Δg_{rel}, radiative corrections Δg_{rad}, recoil corrections Δg_{rec} in (2) were calculated with the accuracy up to terms of order $\alpha^3(m/M)$ and $\alpha^2(m/M)^2$ in [8, 9] on the basis of the quasipotential method for spin 1/2 particles, composing the bound system. The dots designate other possible contributions to the g-factor. In addition, due to the experiments with deuterium, hydrogen-like ions, which have the nucleus of arbitrary spin, there is need to extend the calculational methods of the g-factors on this case. In paper [15] it was suggested an approach for the calculation of the corrections to the gyromagnetic factors, based on the Bargmann-Michel-Telegdi (BMT) equation [16]. The conclusion about independence of the binding corrections on the magnitude of the spin of the constituents was also formulated here. In this work we extended the quasipotential method for the calculation of the magnetic moment of two-particle bound state to the case of arbitrary spin particles and calculated main contributions to the corrections in eq.(2).

The interaction of the massive particles of arbitrary spin with the electromagnetic field is studied on the basis of different methods during a long time [17-25], but still this problem is far from its final solution. It was shown in [18] that the particle of arbitrary spin must have at tree level approximation the gyromagnetic factor g=2. In general case, the matrix element of the electromagnetic current for the particle of arbitrary spin s is determined by means of (2s+1) form factors (charge, magnetic, quadruple, et al.). When studied the magnetic moment of the simple atomic systems, it may be possible to take into account the form factors of the minimal multipolarity, describing the distributions of the electric charge and magnetic moment. The one-particle matrix element J_μ of the

electromagnetic current operator between states with momenta p and q can be written as follows:

$$J_\mu = \bar{U}(p)\left\{\Gamma_\mu F_1^D + \frac{1}{2m}\Sigma_{\mu\nu}k^\nu F_2^P\right\}U(q) \tag{3}$$

The wave function $U(p)$ of particle with arbitrary spin, entering in (3), can be presented in the form (see, for instance [24, 25]):

$$U = \begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} \xi^{\alpha_1\alpha_2\ldots\alpha_p}_{\dot\beta_1\dot\beta_2\ldots\dot\beta_q} \\ \eta^{\beta_1\beta_2\ldots\beta_q}_{\dot\alpha_1\dot\alpha_2\ldots\dot\alpha_p} \end{pmatrix}, \quad p+q=2S, \tag{4}$$

where spin-tensors ξ, η are symmetrical in upper and lower indexes. For the particle of half-integer spin p=S+1/2, q=S-1/2. In the case of integer spin p=q=S. The Lorentz transformation of the spinors ξ and η can be written in the form [25, 26]:

$$\xi = e^{\frac{\vec\Sigma\vec\phi}{2}}\xi_0, \quad \eta = e^{-\frac{\vec\Sigma\vec\phi}{2}}\xi_0, \tag{5}$$

where the direction of the vector $\vec\phi$ coincides with the velocity of the particle, $th\phi = v$. The generator of the Lorentz transformation $\vec\Sigma$ is equal:

$$\vec\Sigma = \sum_{i=1}^{p}\vec\sigma_i - \sum_{i=p+1}^{p+q}\vec\sigma_i, \tag{6}$$

and $\vec\sigma_i$ acts on the ith index of the spinor ξ_0 as follows:

$$\vec\sigma_i\xi_0 = (\vec\sigma_i)_{\alpha_i\beta_i}(\xi_0)_{\ldots\beta_i\ldots}. \tag{7}$$

In the standard representation, which is introduced in analogy with the spin 1/2, the free particle wave function (4) may be written with the accuracy $(v/c)^2$ in the form:

$$U(p) = \begin{pmatrix} \left[1 + \frac{(\vec\Sigma\vec p)^2}{8m^2}\right]\xi_0 \\ \frac{\vec\Sigma\vec p}{2m}\xi_0 \end{pmatrix} \tag{8}$$

The components of the matrix $\Sigma_{\mu\nu}$ in (3) are the generators of the boosts and rotations [25, 26]:

$$\Sigma_{n0} = \begin{pmatrix} \Sigma_n & 0 \\ 0 & -\Sigma_n \end{pmatrix}, \quad \Sigma_{mn} = -2i\epsilon_{mnk}\begin{pmatrix} s_k & 0 \\ 0 & s_k \end{pmatrix}, \quad \vec s = \frac{1}{2}\sum_{i=1}^{2S}\vec\sigma_i. \tag{9}$$

The general expression for the magnetic moment of the two-particle bound system reads as [8, 9]:

$$\vec{\mathcal{M}} = -\frac{i}{2}\left[\frac{\partial}{\partial\vec\Delta}\times < \vec K_A|\vec J(0)|\vec K_B >\right], \quad \vec\Delta = \vec K_A - \vec K_B, \tag{10}$$

where the matrix element of the current operator between the bound states

$$< \vec{K}_A|J_\mu(0)|\vec{K}_B >= \int \frac{d\vec{p}_1 d\vec{p}_2}{(2\pi)^3}\delta(\vec{p}_1+\vec{p}_2-\vec{K}_A)\Psi^*_{\vec{K}_A}(\vec{p})\Gamma_\mu(\vec{p},\vec{q},E_A,E_B)\times \tag{11}$$

$$\times\Psi_{\vec{K}_B}(\vec{q})\delta(\vec{q}_1+\vec{q}_2-\vec{K}_B)\frac{d\vec{q}_1 d\vec{q}_2}{(2\pi)^3},$$

is expressed by means of the wave function of the bound state $\Psi_{\vec{K}_B}(\vec{p})$ and the generalized vertex function Γ_μ presented in Figure 1. The vertex function Γ_μ is determined through the five point function:

$$R_\mu =< 0|\psi_1(t,\vec{x}_1)\psi_2(t,\vec{x}_2)J_\mu(0)\bar{\psi}_1(\tau,\vec{y}_1)\bar{\psi}_2(\tau,\vec{y}_2)|0 >, \tag{12}$$

projected onto the positive-energy states,

$$\Gamma_\mu = G^{-1}R^{(+)}_\mu G^{-1}, \quad R^{(+)}_\mu = U^*_1 U^*_2 R_\mu U_1 U_2, \tag{13}$$

where G is the two-particle Green function. We study the loosely bound composite system, so it is possible to make the perturbation expansion of all quantities Γ, R and G^{-1} in particle interaction.

$$\Gamma = \Gamma^{(0)} + \Gamma^{(1)} + ..., \quad R = R_0 + R_1 + ..., \quad G^{-1} = G^{-1}_0 - V_1 - ... \ , \tag{14}$$

$$\Gamma^{(0)} = G^{-1}_0 R_0 G^{-1}_0, \tag{15}$$

$$\Gamma^{(1)} = G^{-1}_0 R_1 G^{-1}_0 - V_1 G_0 \Gamma^{(0)} - \Gamma^{(0)} G_0 V_1, ... \ , \tag{16}$$

where G_0 is the Green function of two noninteracting particles and V_1 is the one-photon exchange quasipotential (see eq. (19)).

The transformation law for the wave function $\Psi_{\vec{K}_B}(\vec{p})$ of the system of bound particles with spins s_1, s_2 from the rest frame to the reference frame, moving with the momentum $\vec{K}_B$, was obtained in [27]:

$$\delta(\vec{p}_1+\vec{p}_2-\vec{K}_B)\Psi_{\vec{K}_B}(\vec{p}) = D^{S_1}_1(R_W)D^{S_2}_2(R_W)\sqrt{\frac{\epsilon^\circ_1\epsilon^\circ_2 M}{\epsilon_1\epsilon_2 E}}\Psi_0(\vec{p^\circ})\delta(\vec{p^\circ_1}+\vec{p^\circ_2}), \tag{17}$$

where $D^S(R)$ is the well-known rotation matrix and R_W is the Wigner rotation associated with the Lorentz transformation $\Lambda_{\vec{K}_B}$: $(E,\vec{K}_B) = \Lambda_{\vec{K}_B}(M,0)$; $(\epsilon,\vec{p}) = \Lambda(\epsilon^\circ,\vec{p^\circ})$, $E = \sqrt{M^2+\vec{K}^2_B}$, $\epsilon(\vec{p}) = \sqrt{\vec{p}^2+m^2}$. The exact expression for this rotation matrix is the following [9]:

$$D^S(R^W) = S^{-1}(\vec{p})S(\vec{K}_B)S(\vec{p^\circ}), \tag{18}$$

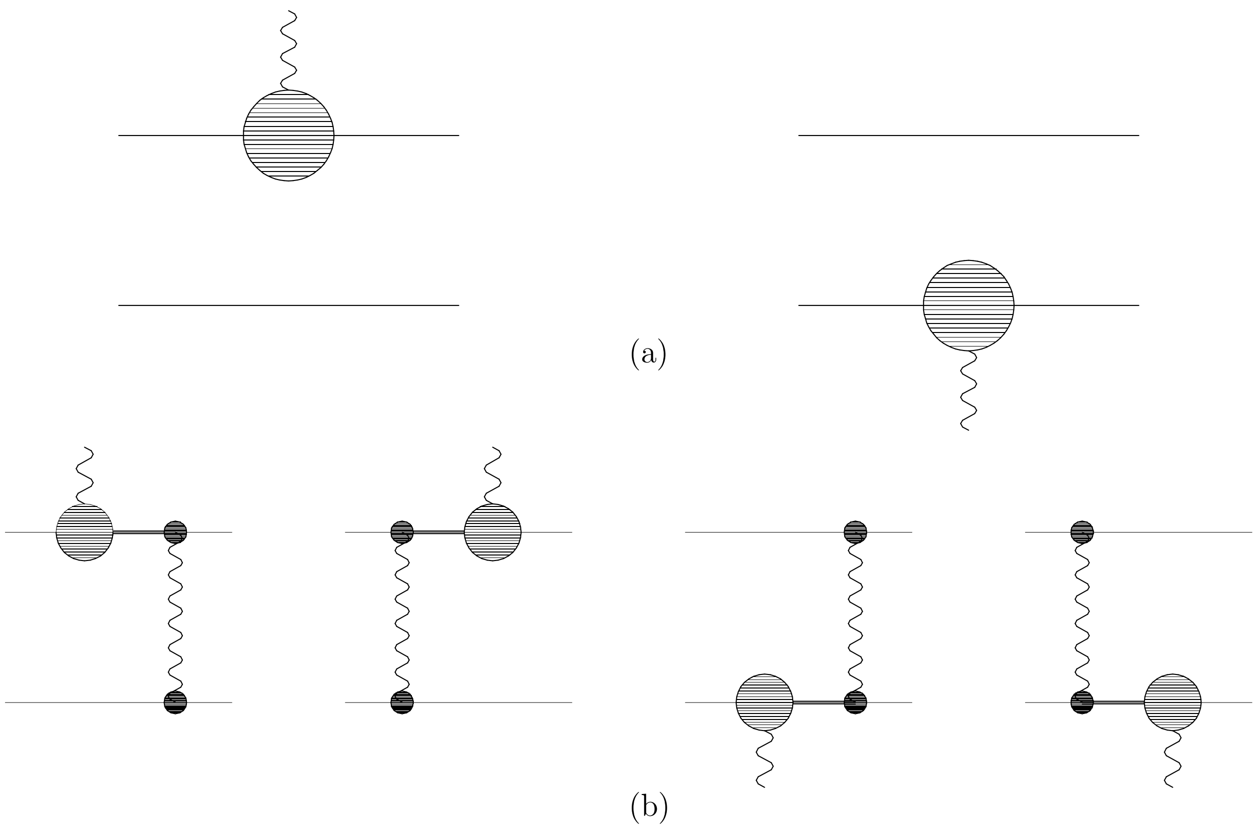

Figure 1: Generalized two-particle vertex function Γ_μ: Diagram (a) represents $\Gamma_\mu^{(0)}$, Diagram (b) describes $\Gamma_\mu^{(1)}$, where the bold line denotes the negative-energy part of the propagator.

where $S(\vec{p})$ is the Lorentz transformation matrix of the spinor wave function (4). The quasipotential bound state wave function $\Psi_0(\vec{p^\circ})$ in the rest frame of the composite system satisfies to the quasipotential equation [28]:

$$G_0^{-1}\Psi \equiv \left(\frac{b^2}{2\mu_R}-\frac{\vec{p^\circ}^2}{2\mu_R}\right)\Psi_0(\vec{p^\circ})=\int V(\vec{p^\circ},\vec{q^\circ},M)\Psi_0(\vec{q^\circ})\frac{d\vec{q^\circ}}{(2\pi)^3}, \tag{19}$$

where the relativistic reduced mass

$$\mu_R=\frac{E_1E_2}{M}=\frac{M^4-(m_1^2-m_2^2)^2}{4M^3}, E_{1,2}=\frac{M^2-m_{2,1}^2+m_{1,2}^2}{2M},$$

$M=E_1+E_2$ is the bound state mass,

$$b^2(M)=\frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2}.$$

In the nonrelativistic limit this equation reduces to the Shroedinger equation with the Coulomb potential. The D^S functions in (18) can be obtained in the approximate form, using (5):

$$D^S(R_W)\approx 1+\frac{\vec{p^\circ}^2-(\vec{\Sigma}\vec{p^\circ})(\vec{\Sigma}\vec{p^\circ})}{4m^2}+\frac{\vec{K}_B^2-(\vec{\Sigma}\vec{K}_B)(\vec{\Sigma}\vec{K}_B)}{4M^2}+\frac{\vec{p^\circ}\vec{K}_B-(\vec{\Sigma}\vec{p^\circ})(\vec{\Sigma}\vec{K}_B)}{4mM}. \tag{20}$$

The main contribution to the vertex function Γ_μ, which is determined by Feynman diagram (a), shown in Figure 1, can be expressed as follows (in the Breit frame):

$$\vec{\Gamma}^{(0)}(\vec{p},\vec{q})=\bar{U}_1(p_1)e_1\left\{\vec{\Gamma}_1+\frac{i\kappa_1}{m_1}\left[\vec{S}_1\times\vec{\Delta}\right]\right\}U_1(q_1)\delta(\vec{p}_2-\vec{q}_2)+(1\leftrightarrow 2), \tag{21}$$

$$\vec{S}_1=\begin{pmatrix}\vec{s}_1 & 0\\ 0 & \vec{s}_1\end{pmatrix},\ \vec{\Delta}=\vec{p}_1-\vec{q}_1,$$

where $F_{1,2}^D(0)=e_{1,2}$, $F_{1,2}^P(0)=e_{1,2}\kappa_{1,2}$, and the matrix

$$\vec{\Gamma}=\begin{pmatrix}0 & \vec{\Sigma}\\ -\vec{\Sigma} & 0\end{pmatrix} \tag{22}$$

is the natural generalization of the corresponding expression of the Dirac matrices $\vec{\gamma}$ for the spin 1/2 in the standard representation. To simplify different terms of eq. (21) it is helpful to use the following relationships [25]:

$$[\Sigma_i,\Sigma_j]=4i\epsilon_{ijk}s_k,\quad [\Sigma_i,s_j]=i\epsilon_{ijk}\Sigma_k. \tag{23}$$

To construct vertex function $\vec{\Gamma}^{(0)}(\vec{p},\vec{q})$, accounting for the terms $(v/c)^2$, we use explicit expression of the wave function (8), transforming the different parts of the matrix element

(21) by means of the equation of motion for the spinor U(p). The following relations are valid (taking into account $\delta(\vec{p}_2-\vec{q}_2)$):

$$\bar{U}_1(\vec{p}_1)\frac{\vec{p}_1+\vec{q}_1}{2m_1}U(\vec{q}_1)=2\vec{p^\circ}-\frac{\epsilon_2}{M}\vec{\Delta}+\frac{i\vec{p^\circ}\left(\vec{S}_1[\vec{p^\circ}\times\vec{\Delta}]\right)}{m_1^2}, \tag{24}$$

$$\bar{U}_1(\vec{p}_1)\frac{\epsilon_1(\vec{p}_1)-\epsilon_1(\vec{q}_1)}{2m_1}\vec{\mathcal{A}}_1U_1(\vec{q}_1)=-\frac{2\vec{p^\circ}\vec{\Delta}}{m_1^2}i[\vec{S}_1\times\vec{p^\circ}], \tag{25}$$

$$\bar{U}_1(\vec{p}_1)[\vec{S}_1\times\vec{\Delta}]U_1(\vec{q}_1)=[\vec{S}_1\times\vec{\Delta}]-\frac{1}{2m_1^2}\left\{\vec{p^\circ}\left(\vec{S}_1[\vec{p^\circ}\times\vec{\Delta}]\right)+[\vec{p^\circ}\times\vec{S}_1](\vec{p^\circ}\vec{\Delta})\right\}. \tag{26}$$

Bound state effects in the vertex function Γ_μ are determined by the diagram (b) of Figure 1. Accounting for iteration terms with the quasipotential, we can represent the corresponding expression in the form [8, 9]:

$$\vec{\Gamma}^{(1)}(\vec{p},\vec{q})=U_1^*(\vec{p}_1)U_2^*(\vec{p}_2)\frac{e_1}{2m_1}\Bigg\{\vec{\mathcal{A}}_1\Lambda_1^{(-)}(\vec{p}_1')\mathcal{B}_1\mathcal{B}_2\hat{V}(\vec{q}_2-\vec{p}_2)+ \tag{27}$$

$$+\mathcal{B}_1\mathcal{B}_2\hat{V}(\vec{p}_2-\vec{q}_2)\Lambda_1^{(-)}(\vec{q}_1')\vec{\mathcal{A}}_1\Bigg\}U_1(\vec{q}_1)U_2(\vec{q}_2)+(1\leftrightarrow 2),$$

$$\hat{V}(\vec{k})=\mathcal{B}_1\mathcal{B}_2\Bigg\{\left(1+\frac{\kappa_1}{2m_1}\vec{\Gamma}_1\vec{k}\right)\left(1-\frac{\kappa_2}{2m_2}\vec{\Gamma}_2\vec{k}\right)- \tag{28}$$

$$-\left(\vec{\mathcal{A}}_1+\frac{\kappa_1}{m_1}\mathcal{B}_1i[\vec{S}_1\times\vec{k}]\right)\left(\vec{\mathcal{A}}_2-\frac{\kappa_2}{m_2}\mathcal{B}_2i[\vec{S}_2\times\vec{k}]\right)\Bigg\}\frac{e_1e_2}{k^2},$$

where the negative-energy projection operator $\Lambda^-(\vec{p})\approx(1-\mathcal{B})/2-\vec{\mathcal{A}}\vec{p}/2m$, $\kappa_{1,2}$ are the anomalous magnetic moments of the particles. The matrices $\vec{\mathcal{A}}_{1,2}$, $\mathcal{B}_{1,2}$ are also the generalizations for $\vec{\alpha}_{1,2}$, $\beta_{1,2}$ used in the case of spin 1/2 particles as in (29):

$$\vec{\mathcal{A}}=\begin{pmatrix}0&\vec{\Sigma}\\ \vec{\Sigma}&0\end{pmatrix},\quad \mathcal{B}=\begin{pmatrix}I&0\\0&-I\end{pmatrix}. \tag{29}$$

Both parts of the quasipotential (28) give contribution to the magnetic moment of the system with the accuracy $(v/c)^2$. Substituting (27), (28) and (8) in (11) and calculating derivative in $\vec{\Delta}$ to eq. (10), we obtain:

$$\vec{\mathcal{M}}=\frac{1}{(2\pi)^3}\int d\vec{p}\Psi_0^*(\vec{p})\frac{e_1}{2\epsilon_1(\vec{p})}\Bigg\{2(1+\kappa_1)\vec{s}_1[1+N_1+N_2]+(1+4\kappa_1)\frac{[\vec{p}\times[\vec{s}_1\times\vec{p}]]}{2m_1^2}+ \tag{30}$$

$$+(1+\kappa_2)\frac{[\vec{p}\times[\vec{s}_2\times\vec{p}]]}{m_1m_2}\frac{\vec{\Sigma}_1^2}{3}-\frac{\epsilon_2(\vec{p})}{M}\left[1+N_1+N_2+\frac{(M-\epsilon_1-\epsilon_2)}{m_2}\frac{\vec{\Sigma}_1^2}{3}\right]i\left[\vec{p}\times\frac{\partial}{\partial\vec{p}}\right]+$$

$$+\frac{1}{2M}\left[\vec{p}\times\left[\vec{p}\times\left(\frac{\vec{s}_1}{m_1}-\frac{\vec{s}_2}{m_2}\right)\right]\right]\Biggr\}\Psi_0(\vec{p})+(1\leftrightarrow 2),$$

where

$$N_i=\frac{\vec{p}^2-(\vec{\Sigma}_i\vec{p})(\vec{\Sigma}_i\vec{p})}{2m_i^2}. \tag{31}$$

In the case of S-states the expression (30) may be essentially simplified:

$$\vec{\mathcal{M}}=\frac{1}{2}g_{1\ bound}\frac{e_1}{m_1}<\vec{s}_1>+\frac{1}{2}g_{2\ bound}\frac{e_2}{m_2}<\vec{s}_2>, \tag{32}$$

where the g-factors of the bound particles are equal:

$$g_{1\ bound}=g_1\Biggl\{1-\frac{<\vec{p}^2>}{3m_1^2}\left[1-\frac{3\kappa_1}{2(1+\kappa_1)}\right]+\frac{<\vec{p}^2>}{2m_1^2}\Biggl[1-\frac{<\vec{\Sigma}_1^2>}{3}+ \tag{33}$$

$$+\frac{m_1^2}{m_2^2}\left(1-\frac{<\vec{\Sigma}_2^2>}{3}\right)\Biggr]+\frac{e_2}{e_1}\frac{<\vec{p}^2>}{3m_2^2}\frac{<\vec{\Sigma}_2^2>}{3}-\frac{<\vec{p}^2>}{(1+\kappa_1)6m_1(m_1+m_2)}\left(1-\frac{e_2}{e_1}\frac{m_1}{m_2}\right)\Biggr\},$$

$$g_{2\ bound}=g_{1\ bound}(1\leftrightarrow 2),\qquad\qquad \frac{1}{2}g_{1,2}=1+\kappa_{1,2}.$$

For the hydrogen-like ion (1 is the electron, 2 is the nucleus) we have: $e_1=-e$, $e_2=Ze$, $<\vec{p}^2>=m_1^2m_2^2(Z\alpha)^2/(m_1+m_2)^2$,

$$K_{s_1}=\frac{<\vec{\Sigma}_1^2>}{3}=1,\ K_{s_2}=\frac{<\vec{\Sigma}_2^2>}{3}=\begin{cases}\frac{4s_2}{3},\ s_2\ \text{is the integer nucleus spin}\\ \frac{4s_2+1}{3},\ s_2\ \text{is the half}-\text{integer nucleus spin}\end{cases}, \tag{34}$$

so the g-factors of the bound electron and nucleus take the form:

$$g_{e\ bound}=g_e\Biggl\{1-\frac{m_2^2(Z\alpha)^2}{3(m_1+m_2)^2}\Biggl[1-\frac{3\kappa_1}{2(1+\kappa_1)}-\frac{3}{2}(1-K_{s_1})- \tag{35}$$

$$-\frac{3}{2}\frac{m_1^2}{m_2^2}\left(1-K_{s_2}-\frac{2}{3}ZK_{s_2}\right)+\frac{m_1}{2(m_1+m_2)(1+\kappa_1)}\left(1+Z\frac{m_1}{m_2}\right)\Biggr]\Biggr\},$$

$$g_{N\ bound}=g_N\Biggl\{1-\frac{m_1^2(Z\alpha)^2}{3(m_1+m_2)^2}\Biggl[1-\frac{3\kappa_2}{2(1+\kappa_2)}-\frac{3}{2}(1-K_{s_2})- \tag{36}$$

$$-\frac{3}{2}\frac{m_2^2}{m_1^2}(1-K_{s_1}-\frac{2}{3Z}K_{s_1})+\frac{m_2}{2(m_1+m_2)(1+\kappa_2)}\left(1+\frac{m_2}{Zm_1}\right)\Biggr]\Biggr\},$$

The expressions (35)-(36) were obtained from interaction amplitudes, shown in Figure 1. They contain corrections of order $O(\alpha^2)$, $O(\alpha^3)$, connected with the bound state effects. As it follows from (30), some of these contributions can be considered as relativistic

corrections for spin s particles (terms proportional to N_i). Other corrections refer to the spin-orbit interaction. Our calculation of relations (35)-(36) shows, that the terms of order $O(\alpha^2)$ in $g_{e\ bound}$, $g_{N\ bound}$ depend on the spin of second particle - nucleus. This conclusion differs from the results of the paper [15], where such dependence is absent. In the case of spin 1/2 particles the obtained expressions (35) and (36) for the bound state g-factors coincide with the results of [8, 10, 11, 20]. The electron g-factors in hydrogen atom, deuterium, tritium as well as their ratios are very important from the experimental point of view [1]. Experimental value of the ratio $g_{e\ H}/g_{e\ D}$, obtained in [13] with high accuracy

$$r^{exp} = \left[\frac{g_{e\ H}}{g_{e\ D}}\right]^{exp} = 1 + 7.22(3) \cdot 10^{-9}. \tag{37}$$

The theoretical expression for this ratio can be written from (35) as follows:

$$r^{th} = \left[\frac{g_{e\ H}}{g_{e\ D}}\right]^{th} = 1 + \alpha^2 \left[\frac{1}{4}\frac{m_1}{m_2} - \frac{25}{72}\frac{m_1^2}{m_2^2} - \frac{\alpha}{\pi}\left(\frac{m_1}{24m_2} - \frac{1}{16}\frac{m_1^2}{m_2^2}\right)\right]. \tag{38}$$

As we pointed out above, the new approach to the calculation of the various order contributions to the magnetic moment of the loosely bound system was suggested in the work [15]. This method is based on the relativistic semiclassical equation of motion for spin. Constructed in [15] on the basis of this equation the interaction Hamiltonian of particles of arbitrary spin with the external electromagnetic field leads to the g-factors of the bound particles, which are independent of its spin. It is known that the BMT equation is approximate one: it is linear on the particle spin, field strength $F_{\mu\nu}$, which doesn't contain coordinate dependence. When the spin s particle is in the bound state in the external homogeneous magnetic field, some terms, omitted in the approximation of the BMT equation, can give definite contribution to the g-factors of the bound particles. In this work during the calculation of nucleus spin-dependent terms in the gyromagnetic factors of the bound particles, composing hydrogen-like ion, we used the approach, proposed in the papers [24, 25], for the description of the electromagnetic interactions of particles with arbitrary spin. New contributions to (35), (36), (38), as compared with [8], were obtained after the replacement of the ordinary boost generators $\vec{\alpha}$ in the case of spin 1/2 particles by operators (29). The value of the correction in r^{th}, connected with the deuteron spin is equal to $\Delta r^{th} = 5\alpha^2 m_1^2/72m_2^2 = 0.001 \cdot 10^{-9}$ (I=1, Z=1, $m_{nucl} = 2m_2$ (m_2 is the proton mass). It lies within the limits of the experimental error, as it follows from eq. (37). Numerically, the ratio (38) $r^{th} = 1 + 7.237 \cdot 10^{-9}$, which is in good agreement with (30). Nucleus spin dependent corrections in (35), (36) are the functions of Z and the number of nucleons in the nucleus N. Despite the increasing of these corrections with Z as Z^3 the growth of the nucleon number N in nucleus leads to opposite effect. So, in the case of ions with nucleous spin I $\neq 0$ the numerical value of these corrections is out of experimental accuracy reached for $^{12}C^{5+}$. For the ion $^{12}C^{5+}$ I=0, so $K_I = 0$ and the corresponding spin-dependent correction is equal to zero. At present time the measurements of

the electron g-factors in the ions $^{16}O^{7+}$ and $^{32}S^{14+}$ were carried out [4], but their nucleus have also spin I=0. From our viewpoint, it will be interesting to measure the values of electron g-factors, using the Penning traps [3, 4] for such ions, which have nucleous spin $I \neq 0$ and the ratio Z^3/N^2 would be reached large values. One of the such ions is the ion $^{59}Co^{26+}$, which has $I = 7/2$, $Z^3/N^2 \approx 5.65$, and the value of spin-dependent correction in (35) is equal to $0.1 \cdot 10^{-9}$. As was pointed out by W.Quint [4], the measurement of the bound electron g-factors with the accuracy exceeding 1 ppb may be realized in the near future.

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